Mathematical programming in an auction to provide Internet connections in Buenos Aires City schools

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Contents

Combinatorial Auctions

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- These models comprise many aspects, including the auction design, the development of mathematical models for determining the auction outcome, and the implementation of suitable algorithms for solving these models.
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Combinatorial Auctions

- Since these problems are interdisciplinary, their formulation and solution involves economists, management experts, operations researchers, game theorists, and computer scientists.
- A book surveying the state of the art is *"Combinatorial Auctions"*, Peter Cramton, Yoav Shoham, and Richard Steinberg (2006), MIT Press.
- A successful case:
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A successful case: JUNAEB

- The National Bureau of Help and Grants for Schools (JUNAEB) in Chile provides meals to around ten thousand schools in Chile.
- Different kinds of meals are supplied for more than two million children at school ages.
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- For practical purposes, each of the 13 regions (provinces) in Chile is subdivided in Territorial Units (TU). There are over 100 TUs.
- Each company taking part in the auction presents bids for groups of TUs. Each bid for a group of TUs is either accepted or not, thus taking advantage of economies of scale.
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- Since 1997, a mathematical model developed by the University of Chile has been regularly applied, in order to support the decision-making process. The model objective consists in finding the set of bids covering all the auctioned TUs and minimizing the total expenses.
- Thanks to the implementation of this model, savings of over 40 million dollars per year have been attained.

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Our problem

Auction the Internet connection service for the 709 school buildings in Buenos Aires City, among the interested Internet providers.

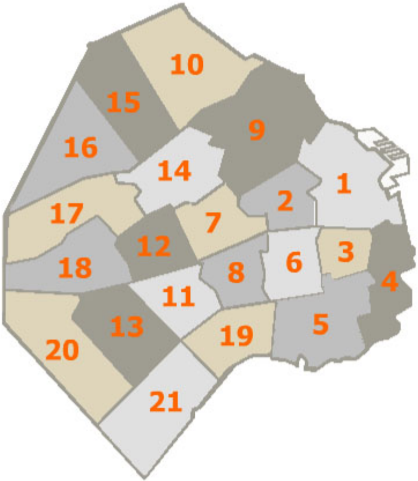
Original City Government proposal

- Auction each school separately.
- Each Internet provider makes a bid for each school.
- Each school is assigned the Internet provider offering the best price for that school.
- Problems: no economies of scale captured; possibility of collusion; possible price distortion (higher prices for schools in areas with little competence).

Our first proposal

- Basic units: the School Districts.
- The companies bid for groups of districts (as in the JUNAEB auction).
- Main problem: the coverage areas of the companies do not conform to the borders of the School Districts (it is much more expensive to install the technology outside the actual coverage area), and bids would be affected.

School Districts



Coverage areas of the potential bidders

Cablevision

=

Multicanal

Coverage areas of the potential bidders

Telefónica

Coverage areas of the potential bidders

Telecom

Coverage areas of the potential bidders

Telmex

Coverage areas of the potential bidders

iPlan

Coverage areas of the potential bidders

Metrotel

Our second proposal

- Design new territorial units taking into account the coverage areas.

- Problems:
  1. some coverage areas go across these new TUs,
  2. possibility of collusion in the TUs with only two competing companies.

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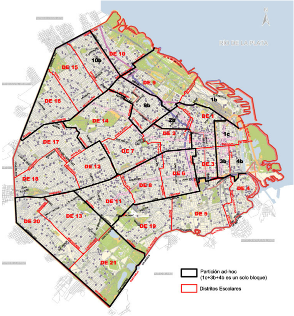

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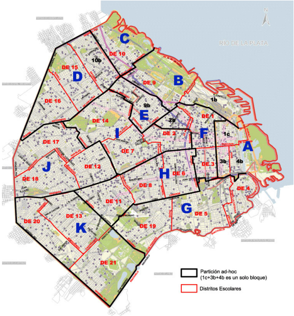

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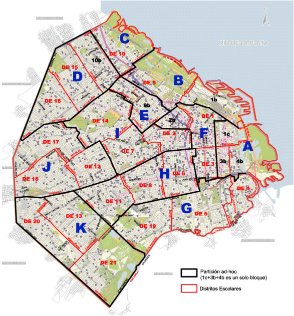


- Problems:
  1. some coverage areas go across these new TUs,
  2. possibility of collusion in the TUs with only two competing companies.

Our third proposal

- Each company provides a price for a single school (the same for all schools).
- Each company provides a list of schools it can serve.
- Each company provides volume discounts (with prespecified price intervals).
- Upper bound on the number of schools each company can receive (proposal finally not accepted).

Our third proposal

- Benefits:
  1. Almost impossible to collude.
  2. A company cannot impose a high price in areas with low competence, and propose lower prices in areas with higher competence.
  3. Economies of scale are captured.
  4. The company itself defines its coverage area through the list of schools it can serve.
- Observation: this was the chosen model. Note that this is not properly a “combinatorial auction” but a “multi-unit auction”, so we refer to an application of mathematical programming to auctions.

Foundations of the mathematical model

- The optimal solution for the city is obtained, assigning an Internet provider to each school.
- The territorial units are designed after the bids are presented, in terms of the lists of schools provided by the companies (note that this formulation is more efficient than the natural formulation with one binary variable per school and company, which has much more variables).
- The model solution determines how many (and not which) schools each company receives at each territorial unit (each unit it not neccessarily assigned as a whole).
- After the model is solved, if there are territorial units shared by two or more companies, a manual or automatic process can be applied to assign the schools to the companies (for example, by following geographically based criteria).

The integer programming model

Model parameters:

- C: set of Internet providers/companies
- R: set of territorial units/regions, defined by the intersection of the companies' coverage areas
- $E_r, \ r \in R$: set of schools in the region r
- $p_{ji}, \ j \in R, \ i \in C$: 1 if the company i bids in the region j, 0 otherwise
- T: set of price intervals (0–19, 20–39, . . . , 80–99, 100–149, 150–199, 200–299, . . . , 500–600, 600–709)
- $\text{mín}(t)$ and $\text{máx}(t)$, $t \in T$, lower and upper bounds of each price interval
- $c_{ti}, \ t \in T, \ i \in C$: cost per school in the price interval t offered by the company i

The integer programming model

Model variables:

- $x_{ji} \in \mathbb{Z}_{\geq 0},\ j \in R,\ i \in C$: number of schools in the region j assigned to the company i
- $y_{it} \in \{0, 1\},\ i \in C,\ t \in T$: specifies whether the company i is using the price interval t
- $z_{it} \in \mathbb{Z}_{\geq 0},\ i \in C,\ t \in T$: number of schools assigned to the company i in the price interval t. It can be defined in $\mathbb{R}_{\geq 0}$, since in the optimal solution this variable will be set at an integer value.

The integer programming model

$$\text{mín} \sum_{i\in C}\sum_{t\in T} c_{ti} z_{it}$$

$$\begin{aligned}
\sum_{i\in C} x_{ji} &= |E_j| \qquad \forall\, j \in R \\
\sum_{j\in R} x_{ji} &\geq \text{mín}(t) - M(1-y_{it}) \qquad \forall\, i \in C,\ \forall\, t \in T \\
\sum_{j\in R} x_{ji} &\leq \text{máx}(t) + M(1-y_{it}) \qquad \forall\, i \in C,\ \forall\, t \in T \\
\sum_{t\in T} y_{it} &= 1 \qquad \forall\, i \in C \\
z_{it} &\geq \left(\sum_{j\in R} x_{ji}\right) - M(1-y_{it}) \qquad \forall\, i \in C,\ \forall\, t \in T \\
x_{ji} &\leq p_{ji} \qquad \forall\, i \in C,\ \forall\, j \in R \\
\sum_{t\in T} z_{it} &= \sum_{j\in R} x_{ji} \qquad \forall\, i \in C
\end{aligned}$$

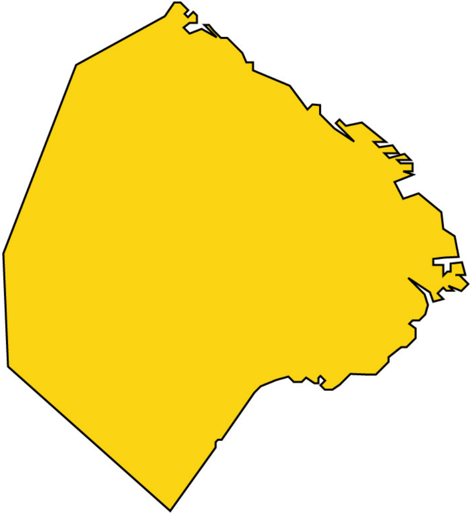
Searching for multiple optima

- Let $x_{ij} = a_{ij}$ be the optimal solution, for every company i and region j.
- For each such equation, we add two binary variables w_{ij} and w'_{ij}. Furthermore, we add the following constraints:

$$
\begin{aligned}
x_{ij} &\geq (a_{ij}+1)w_{ij} \qquad \forall ij : a_{ij} \neq 0 \\
709 - x_{ij} &\geq (709 - (a_{ij}-1))w'_{ij} \qquad \forall ij : a_{ij} \neq 0 \\
\sum_{a_{ij} \neq 0} (w_{ij} + w'_{ij}) &= 1
\end{aligned}
$$

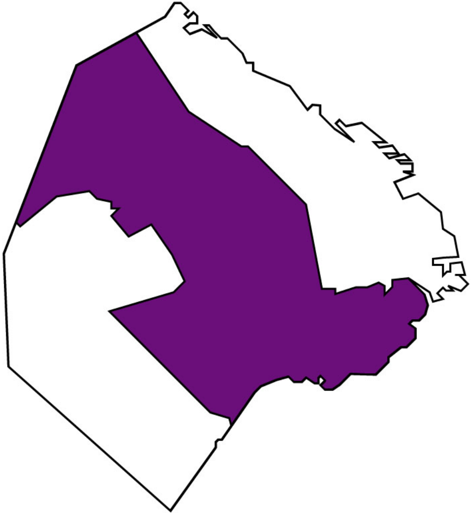
Received bids

Cablevisión
709 schools
(whole set of
schools)



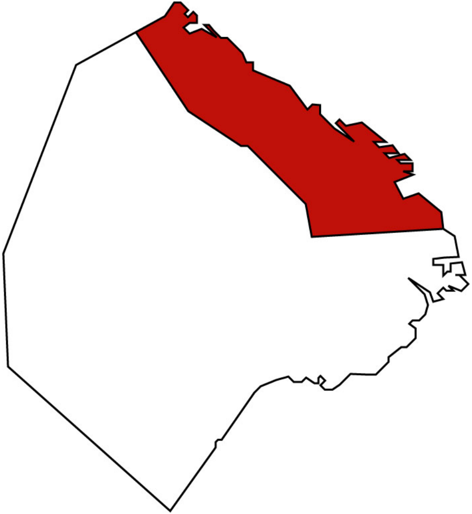
Received bids

Telefónica
348 schools



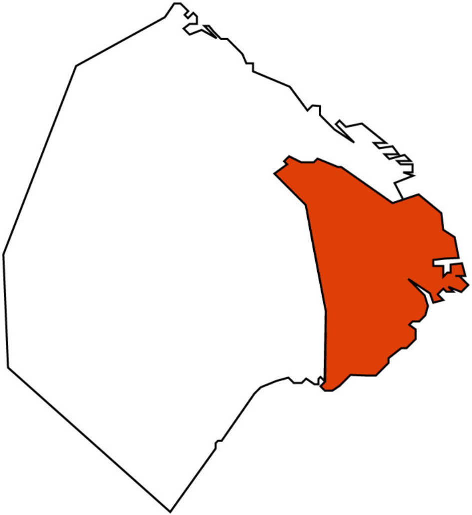
Received bids

Telecom
99 schools

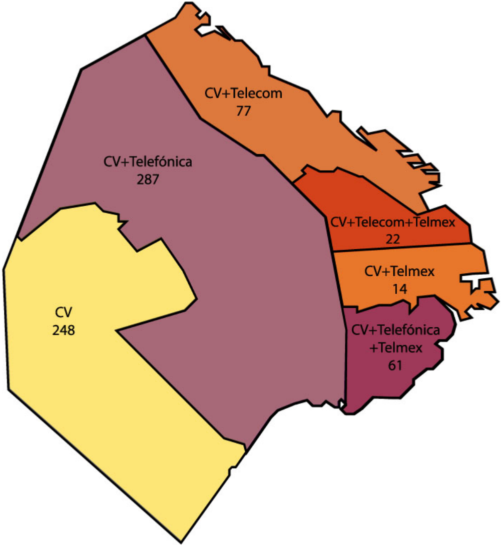


Received bids

Telmex
97 schools



Territorial units/regions defined by the bids

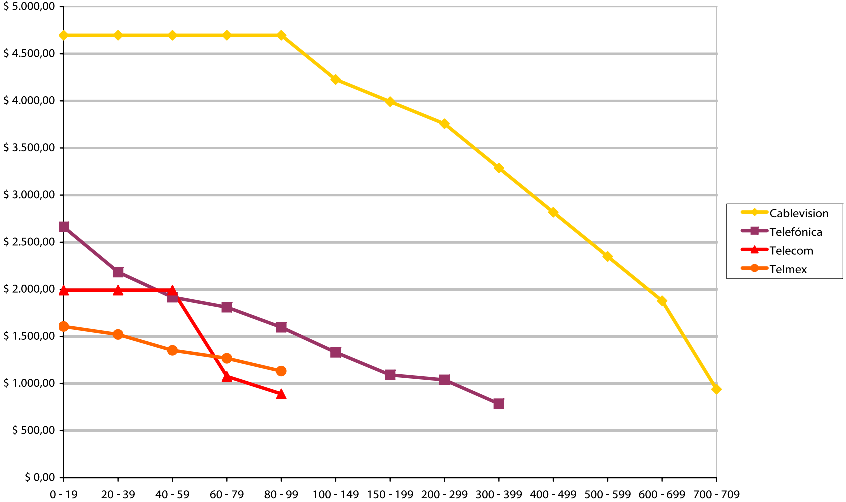


Bids: Individual price as a function of the price interval

	Cablevision		Telefónica		Telecom		Telmex	
Tramo	Descuento	Precio unitario	Descuento	Precio unitario	Descuento	Precio unitario	Descuento	Precio unitario
0 - 19	0%	$ 4.696,70	0%	$ 2.662,00	0%	$ 1.991,66	5%	$ 1.605,50
20 - 39	0%	$ 4.696,70	18%	$ 2.182,84	0%	$ 1.991,66	10%	$ 1.521,00
40 - 59	0%	$ 4.696,70	28%	$ 1.916,64	0%	$ 1.991,66	20%	$ 1.352,00
60 - 79	0%	$ 4.696,70	32%	$ 1.810,16	33%	$ 1.075,50	25%	$ 1.267,50
80 - 99	0%	$ 4.696,70	40%	$ 1.597,20	45.02%	$ 890,07	33%	$ 1.132,30
100 - 149	10%	$ 4.227,03	50%	$ 1.331,00	---	---	---	---
150 - 199	15%	$ 3.992,20	59%	$ 1.091,42	---	---	---	---
200 - 299	20%	$ 3.757,36	61%	$ 1.038,18	---	---	---	---
300 - 399	30%	$ 3.287,69	70.5%	$ 785,29	---	---	---	---
400 - 499	40%	$ 2.818,02	---	---	---	---	---	---
500 - 599	50%	$ 2.348,36	---	---	---	---	---	---
600 - 699	60%	$ 1.878,68	---	---	---	---	---	---
700 - 709	80%	$ 939,34	---	---	---	---	---	---

- The prices of Cablevisión and Telmex are multiplied by 0.9 since these companies provide connections based on fiber optics (such calculation was prespecified in the auction terms). Telefónica and Telecom provide connections based on copper wires.

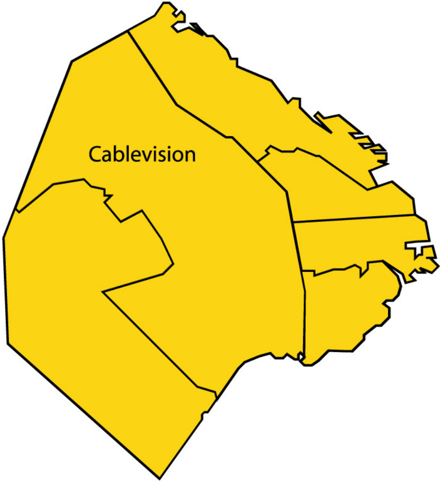
Bids: Individual price as a function of the price interval



Results

Scenario 1: All the companies take part in the process

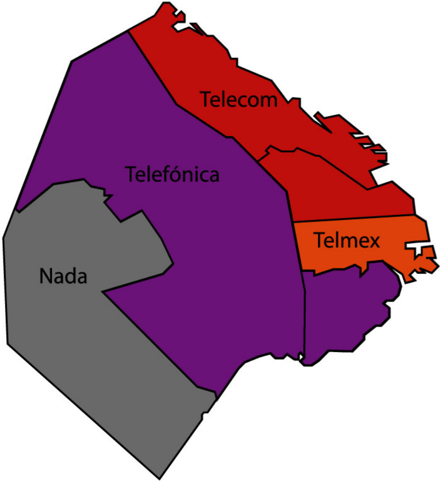
- Cablevision wins all 709 schools
- Total cost: $ 665.992,06
- Average cost per school: $ 939,34



Results

Scenario 2: Cablevision does not take part in the process (since there existed a formal protest against this company)

- 461 schools are assigned.
  - # of schools assigned to Telefónica: 348.
  - # of schools assigned to Telecom: 99.
  - # of schools assigned to Telmex: 14.
- Total cost: $ 382.691,85
- Average cost per school: $ 830,13



Conclusions

- Our main contribution was the design of the auction process since, due to the number and properties of the received bids, the optimal assignment was not difficult to find.
- By analyzing the Scenario 1 (final price: $ 665.992,06), we can estimate the overall benefit by making reasonable conjectures on the bids that the companies would have made in a school-per-school auction.

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Conclusions

- The cost is $ 1.539.330,21 assuming that in the schools where CV is the only bidder, this company offers its worst price ($ 4696,7), and that in the remaining schools each company offers its best price. Note that this assumption is not unlikely to hold, since CV used its monopoly to win all 709 schools with an average price which is higher than the best received price, hence it could have used this monopoly in order to charge a high price in the schools where it is the only bidder.
- Savings: $ 1.539.330,21 - $ 665.992,06 = $ 873.338,15

Conclusions

- The cost is $ 840.461,25 making the *very optimistic* assumption that in the schools where CV is the only bidder, this company offers its second best price ($ 1878,68), and in the remaining schools each company offers its best price.
- Savings: $ 840.461,25 - $ 665.992,06 = $ 174.469,19

Conclusions

- The cost is \$ 827.931,33 making the *even more optimistic* assumption that in every school with two or more bidders, some company offers the best received price (\$ 785,29) and that CV offers its second best price (\$ 1878,68) in the schools where it is the only bidder.
- Savings: \$ 827.931,33 - \$ 665.992,06 = \$ 161.939,27